Components of a Force

Forces acting at some angle from the the coordinate axes can be resolved into mutually perpendicular forces called *components*. The component of a force parallel to the x-axis is called the x-component, parallel to y-axis the y-component, and so on.

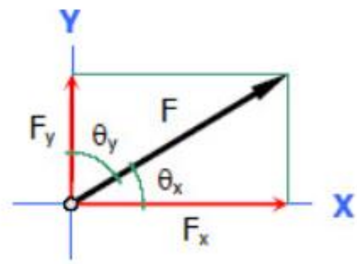
Components of a Force in XY Plane

$$F_x = F\cos\theta_x = F\sin\theta_y$$

$$F_y = F\sin\theta_x = F\cos\theta_y$$

$$F = \sqrt{F_x{}^2 + F_y{}^2}$$

$$\tan\theta_x = \frac{F_y}{F_x}$$

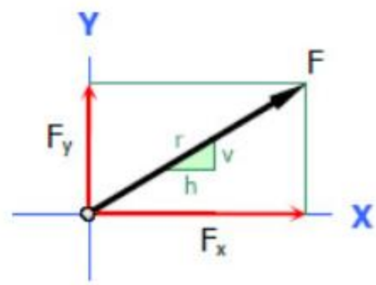


Given the slope of the line of action of the force as v/h (see figure to the right)

$$r = \sqrt{h^2 + v^2}$$

$$F_x = F(h/r)$$

$$F_y = F(v/r)$$



Components of a Force in 3D Space

Given the *direction cosines* of the force:

$$F_x = F\cos\theta_x$$

$$F_y = F\cos\theta_y$$

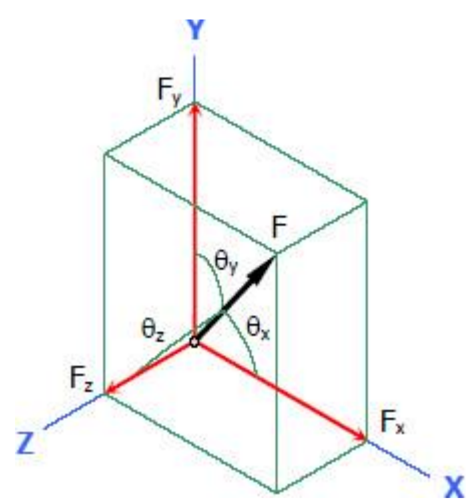
$$F_z = F\cos\theta_z$$

$$F = \sqrt{F_x{}^2 + F_y{}^2 + F_z{}^2}$$

$$\cos\theta_x = \frac{F_x}{F}$$

$$\cos\theta_y = \frac{F_y}{F}$$

$$\cos\theta_z = \frac{F_z}{F}$$



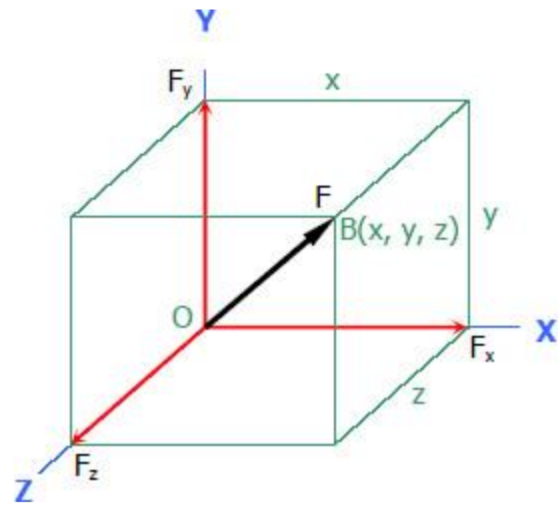
Given the coordinates of any two points along the line of action of the force (in reference to the figure shown, one of the points is the origin):
Let d = distance OB

$$d = \sqrt{x^2 + y^2 + z^2}$$

$$F_x = F(x/d)$$

$$F_y = F(y/d)$$

$$F_z = F(z/d)$$



Vector Notation of a Force (Rectangular Representation of a Force)

$$F = F\lambda$$

Where λ is a unit vector. There are two cases in determining λ; by direction cosines and by the coordinates of any two points on the line of action of the force.

Given the direction cosines:
$\lambda = \cos\theta_x i + \cos\theta_y j + \cos\theta_z k$
Given any two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ on the line of action of the force:
$\lambda = 1/d(d_x i + d_y j + d_z k)$
Where
i, **j**, and **k** are unit vectors in the direction of x, y and z respectively.
$d_x = x_2 - x_1$
$d_y = y_2 - y_1$
$d_z = z_2 - z_1$

$$d = \sqrt{{d_x}^2 + {d_y}^2 + {d_z}^2}$$

$$\cos\theta_x = d_x/d$$
$$\cos\theta_y = d_y/d$$
$$\cos\theta_z = d_z/d$$

Note:

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$$
$$(d_x/d)^2 + (d_y/d)^2 + (d_z/d)^2 = 1$$

Also note the following:

$$F_x = \cos\theta_x = d_x/d$$
$$F_y = \cos\theta_y = d_y/d$$
$$F_z = \cos\theta_z = d_z/d$$

$$F = \sqrt{{F_x}^2 + {F_y}^2 + {F_z}^2}$$

Thus,

$$F=F(\cos\theta_x i+\cos\theta_y j+\cos\theta_z k)$$
$$F=F/d(dxi+dyj+dzk)$$

In simplest term

$$F=F_x i+F_y j+F_z k$$

Problem 001

Problem Determine the x and y components of the forces shown below in Fig P

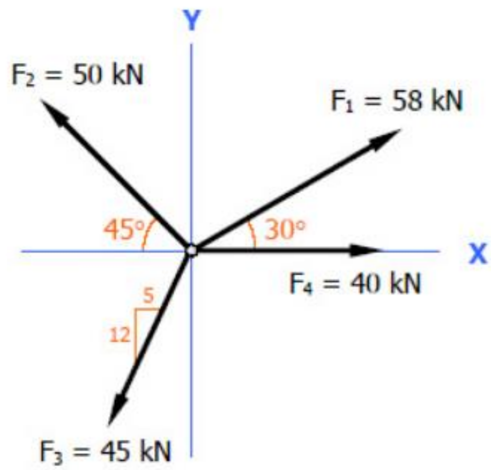


Figure P-001

Solution 001

$F_{x1} = 58\cos 30° = 50.23\ \text{kN}$

$F_{y1} = 58\sin 30° = 29\ \text{kN}$

$F_{x2} = -50\cos 45° = -35.36\ \text{kN}$

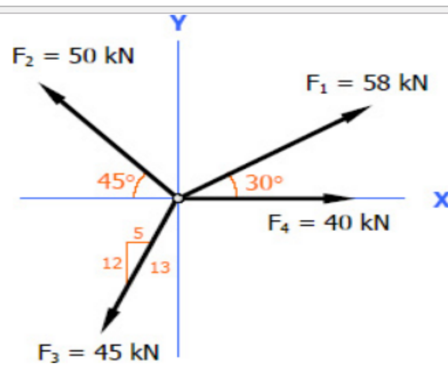
$F_{y2} = 50\sin 45° = 35.36\ \text{kN}$

$F_{x3} = -45(\frac{5}{13}) = -17.31\ \text{kN}$

$F_{y3} = -45(\frac{12}{13}) = -41.54\ \text{kN}$

$F_{x4} = 40\ \text{kN}$

$F_{y4} = 0$



Rectangular Representation

$\mathbf{F} = F(\cos\theta_x \mathbf{i} + \sin\theta_x \mathbf{j})$

$\mathbf{F_1} = 58(\cos 30°\mathbf{i} + \sin 30°\mathbf{j}) = 50.23\mathbf{i} + 29\mathbf{j}\ \text{kN}$

$\mathbf{F_2} = 50(-\cos 45°\mathbf{i} + \sin 45°\mathbf{j}) = -35.36\mathbf{i} + 35.36\mathbf{j}\ \text{kN}$

$\mathbf{F_3} = 45(-\frac{5}{13}\mathbf{i} - \frac{12}{13}\mathbf{j}) = -17.31\mathbf{i} - 41.54\mathbf{j}\ \text{kN}$

$\mathbf{F_4} = 40\mathbf{i}\ \text{kN}$

From the above vector notations, F_x is the coefficient of **i** and F_y is the coefficient of **j**.

Problem 002

Compute the x and y components of each of the four forces shown in Fig. P-002.

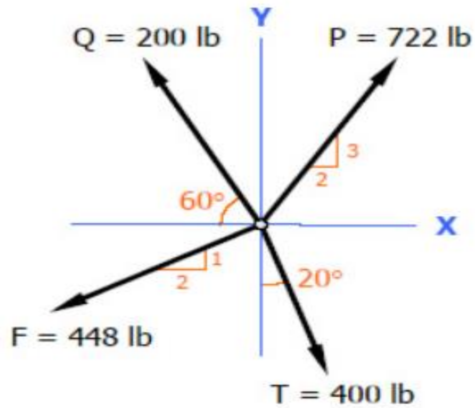
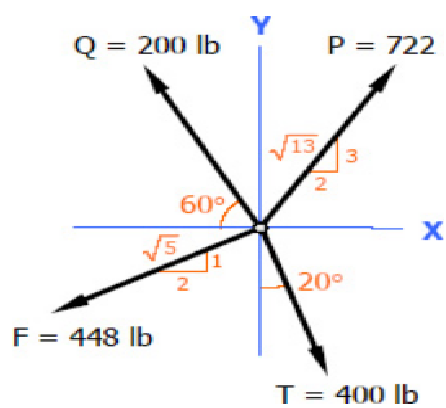


Figure P-002

Solution 002



$$P_x = 722\left(\frac{2}{\sqrt{13}}\right) = 400.49 \text{ lb}$$

$$P_y = 722\left(\frac{3}{\sqrt{13}}\right) = 600.74 \text{ lb}$$

$$Q_x = -200\cos 60^\circ = -100 \text{ lb}$$

$$Q_y = 200\sin 60^\circ = 173.20 \text{ lb}$$

$$F_x = -448\left(\frac{2}{\sqrt{5}}\right) = -400.70 \text{ lb}$$

$$F_y = -448\left(\frac{1}{\sqrt{5}}\right) = -200.35 \text{ lb}$$

$$T_x = 400\sin 20^\circ = 136.81 \text{ lb}$$

$$T_y = -400\cos 20^\circ = -375.88 \text{ lb}$$

Rectangular Representation

$$\mathbf{F} = F(\cos\theta_x \mathbf{i} + \sin\theta_x \mathbf{j})$$

$$\mathbf{P} = 722\left(\frac{2}{\sqrt{13}}\mathbf{i} + \frac{3}{\sqrt{13}}\mathbf{j}\right) = 400.49\mathbf{i} + 600.74\mathbf{j}$$

$$T_y = -400\cos 20^\circ = -375.88 \text{ lb}$$

Rectangular Representation

$$\mathbf{F} = F(\cos\theta_x \mathbf{i} + \sin\theta_x \mathbf{j})$$

$$\mathbf{P} = 722\left(\frac{2}{\sqrt{13}}\mathbf{i} + \frac{3}{\sqrt{13}}\mathbf{j}\right) = 400.49\mathbf{i} + 600.74\mathbf{j} \text{ lb}$$

$$\mathbf{Q} = 200(-\cos 60^\circ \mathbf{i} + \sin 60^\circ \mathbf{j}) = -100\mathbf{i} + 173.20\mathbf{j} \text{ lb}$$

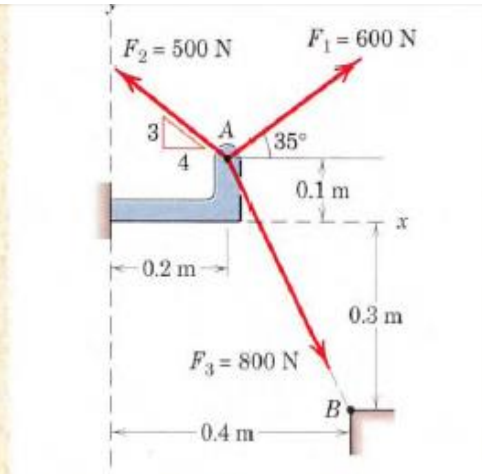
$$\mathbf{F} = 448\left(-\frac{2}{\sqrt{5}}\mathbf{i} - \frac{1}{\sqrt{5}}\mathbf{j}\right) = -400.70\mathbf{i} - 200.35\mathbf{j} \text{ lb}$$

$$\mathbf{T} = 400(\sin 20^\circ \mathbf{i} - \cos 20^\circ \mathbf{j}) = 136.81\mathbf{i} - 375.88\mathbf{j} \text{ lb}$$

The coefficients of **i** and **j** from the vector notations are the respective x and y components of each force.

Problem 3

The forces $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$, all of which act on point A of the bracket, are specified in three different ways. Determine the x and y scalar components of each of the three forces.



Solution. The scalar components of $\mathbf{F}_1$, from Fig. a, are

$$F_{1_x} = 600 \cos 35^\circ = 491 \text{ N} \qquad \textit{Ans.}$$

$$F_{1_y} = 600 \sin 35^\circ = 344 \text{ N} \qquad \textit{Ans.}$$

The scalar components of $\mathbf{F}_2$, from Fig. b, are

$$F_{2_x} = -500(\tfrac{4}{5}) = -400 \text{ N} \qquad \textit{Ans.}$$

$$F_{2_y} = 500(\tfrac{3}{5}) = 300 \text{ N} \qquad \textit{Ans.}$$

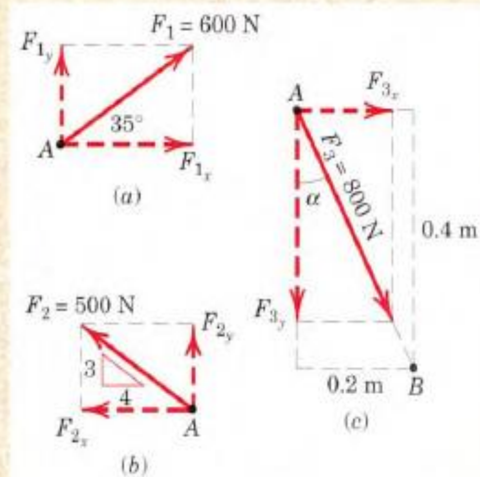
Note that the angle which orients $\mathbf{F}_2$ to the x-axis is never calculated. The cosine and sine of the angle are available by inspection of the 3-4-5 triangle. Also note that the x scalar component of $\mathbf{F}_2$ is negative by inspection.

The scalar components of $\mathbf{F}_3$ can be obtained by first computing the angle α of Fig. c.

$$\alpha = \tan^{-1}\left[\frac{0.2}{0.4}\right] = 26.6^\circ$$

Then $F_{3_x} = F_3 \sin \alpha = 800 \sin 26.6^\circ = 358 \text{ N}$ *Ans.*

$$F_{3_y} = -F_3 \cos \alpha = -800 \cos 26.6^\circ = -716 \text{ N} \qquad \textit{Ans.}$$



Many problems in mechanics require analysis in three dimensions, and for such problems it is often necessary to resolve a force into its three mutually perpendicular components. The force **F** acting at point O in Fig. 2/16 has the *rectangular components* F_x, F_y, F_z, where

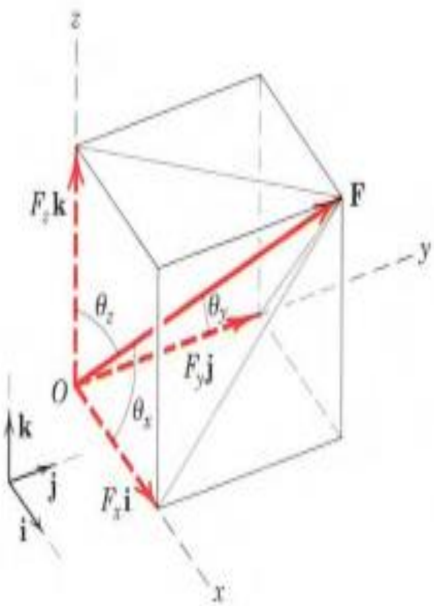


Figure 2/16

$$\boxed{\begin{aligned} F_x &= F\cos\theta_x & F &= \sqrt{F_x^{\,2} + F_y^{\,2} + F_z^{\,2}} \\ F_y &= F\cos\theta_y & \mathbf{F} &= F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k} \\ F_z &= F\cos\theta_z & \mathbf{F} &= F(\mathbf{i}\cos\theta_x + \mathbf{j}\cos\theta_y + \mathbf{k}\cos\theta_z) \end{aligned}} \qquad (2/11)$$

The unit vectors **i**, **j**, and **k** are in the x-, y-, and z-directions, respectively. Using the direction cosines of **F**, which are $l = \cos\theta_x$, $m = \cos\theta_y$, and $n = \cos\theta_z$, where $l^2 + m^2 + n^2 = 1$, we may write the force as

$$\boxed{\mathbf{F} = F(l\mathbf{i} + m\mathbf{j} + n\mathbf{k})} \qquad (2/12)$$

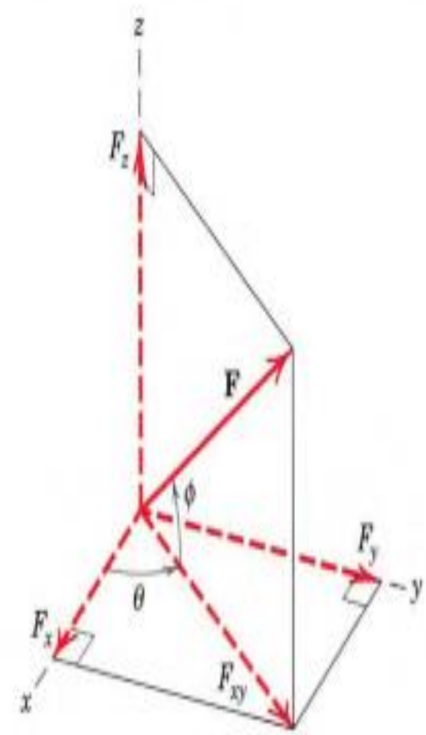
(b) Specification by two angles which orient the line of action of the force. Consider the geometry of Fig. 2/18. We assume that the angles θ and ϕ are known. First resolve **F** into horizontal and vertical components.

$$F_{xy} = F\cos\phi$$
$$F_z = F\sin\phi$$

Then resolve the horizontal component F_{xy} into x- and y-components.

$$F_x = F_{xy}\cos\theta = F\cos\phi\cos\theta$$
$$F_y = F_{xy}\sin\theta = F\cos\phi\sin\theta$$

The quantities F_x, F_y, and F_z are the desired scalar components of **F**.



Problem 003

Which of the following correctly defines the 500 N force that passes from A(4, 0, 3) to B(0, 6, 0)?

A. 256**i** - 384**j** + 192**k** N

B. -256**i** + 384**j** - 192**k** N

C. -384**i** + 192**j** - 256**k** N

D. 384**i** - 192**j** + 256**k** N

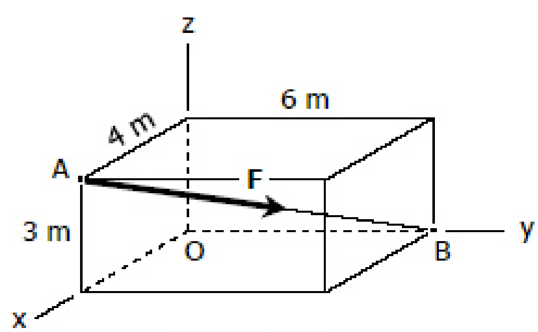


Figure 1.3

From the figure

$$\mathbf{r}_{AB} = -4\mathbf{i} + 6\mathbf{j} - 3\mathbf{k} \ \text{ m}$$

Unit vector from A to B:

$$\lambda_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}}$$

$$\lambda_{AB} = \frac{-4\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}}{\sqrt{(-4)^2 + 6^2 + (-3)^2}}$$

$$\lambda_{AB} = -0.5121\mathbf{i} + 0.7682\mathbf{j} - 0.3841\mathbf{k}$$

Rectangular representation of **F**:

$$\mathbf{F} = F\,\lambda_{AB}$$

$$\mathbf{F} = 500(-0.5121\mathbf{i} + 0.7682\mathbf{j} - 0.3841\mathbf{k})$$

$$\mathbf{F} = -256\mathbf{i} + 384\mathbf{j} - 192\mathbf{k} \ \text{ N}$$

Answer: **B**

Problem 004

Referring to Fig. 1.4, determine the angle between vector A and the y-axis.

A. 65.7°

B. 73.1°

C. 67.5°

D. 71.3°

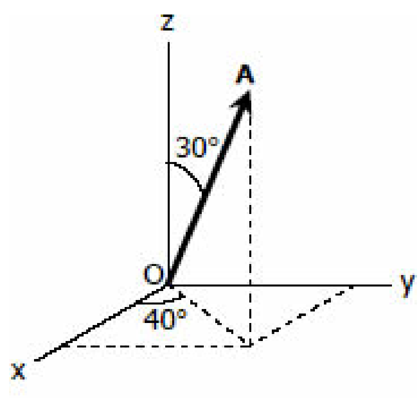


Figure 1.4

Solution 004

$$A_{xy} = A \sin 30^\circ$$

$$A_{xy} = 0.5A$$

$$A_y = A_{xy} \sin 40^\circ$$

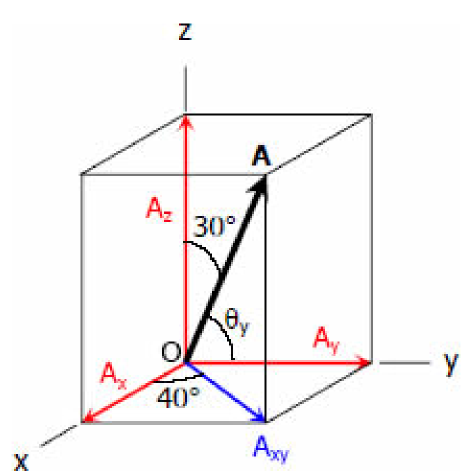
$$A_y = (0.5A) \sin 40^\circ$$

$$A_y = 0.321A$$

$$\cos \theta_y = 0.321$$

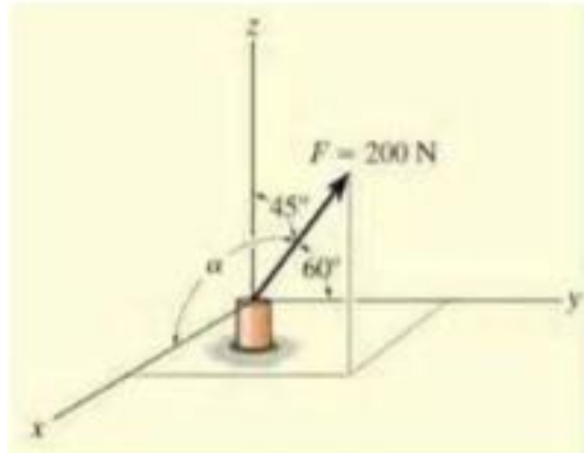
$$\theta_y = 71.3^\circ$$

Answer: **D**



Example 8: Express the $\boldsymbol{F}$ shown in Figure as a Cartesian components

Solution: Since only two coordinate direction angles are specified, the third angle α must be determined from the equation;



$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\cos^2\alpha + \cos^2 60^\circ + \cos^2 45^\circ = 1$$

$$\cos\alpha = \sqrt{1 - \cos^2 60^\circ - \cos^2 45^\circ} = \pm 0.5$$

Hence two possibilities exist, namely

$$\alpha = \cos^{-1}(0.5) = 60^\circ \quad \text{or} \quad \alpha = \cos^{-1}(-0.5) = 120^\circ$$

By inspection it is necessary that $\alpha = 60^\circ$, since $\boldsymbol{F}_x$ must be in $+x$ direction.

$$F_x = F\cos\alpha = 200\cos 60 = 100\text{N}$$

$$F_y = F\cos\beta = 200\cos 60 = 100\text{N}$$

$$F_z = F\cos\gamma = 200\cos 45 = 141.4\text{N}$$

Express the force **F** shown in Fig. 2–32*a* as a Cartesian vector.

SOLUTION

The angles of 60° and 45° defining the direction of **F** are *not* coordinate direction angles. Two successive applications of the parallelogram law are needed to resolve **F** into its *x, y, z* components. First $\mathbf{F} = \mathbf{F}' + \mathbf{F}_z$, then $\mathbf{F}' = \mathbf{F}_x + \mathbf{F}_y$, Fig. 2–32*b*. By trigonometry, the magnitudes of the components are

$$F_z = 100 \sin 60° \text{ lb} = 86.6 \text{ lb}$$

$$F' = 100 \cos 60° \text{ lb} = 50 \text{ lb}$$

$$F_x = F' \cos 45° = 50 \cos 45° \text{ lb} = 35.4 \text{ lb}$$

$$F_y = F' \sin 45° = 50 \sin 45° \text{ lb} = 35.4 \text{ lb}$$

Realizing that $\mathbf{F}_y$ has a direction defined by $-\mathbf{j}$, we have

$$\mathbf{F} = \{35.4\mathbf{i} - 35.4\mathbf{j} + 86.6\mathbf{k}\} \text{ lb} \qquad \textit{Ans.}$$

To show that the magnitude of this vector is indeed 100 lb, apply Eq. 2–4,

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$= \sqrt{(35.4)^2 + (35.4)^2 + (86.6)^2} = 100 \text{ lb}$$

If needed, the coordinate direction angles of **F** can be determined from the components of the unit vector acting in the direction of **F**. Hence,

$$\mathbf{u} = \frac{\mathbf{F}}{F} = \frac{F_x}{F}\mathbf{i} + \frac{F_y}{F}\mathbf{j} + \frac{F_z}{F}\mathbf{k}$$

$$= \frac{35.4}{100}\mathbf{i} - \frac{35.4}{100}\mathbf{j} + \frac{86.6}{100}\mathbf{k}$$

$$= 0.354\mathbf{i} - 0.354\mathbf{j} + 0.866\mathbf{k}$$

so that

$$\alpha = \cos^{-1}(0.354) = 69.3°$$

$$\beta = \cos^{-1}(-0.354) = 111°$$

$$\gamma = \cos^{-1}(0.866) = 30.0°$$

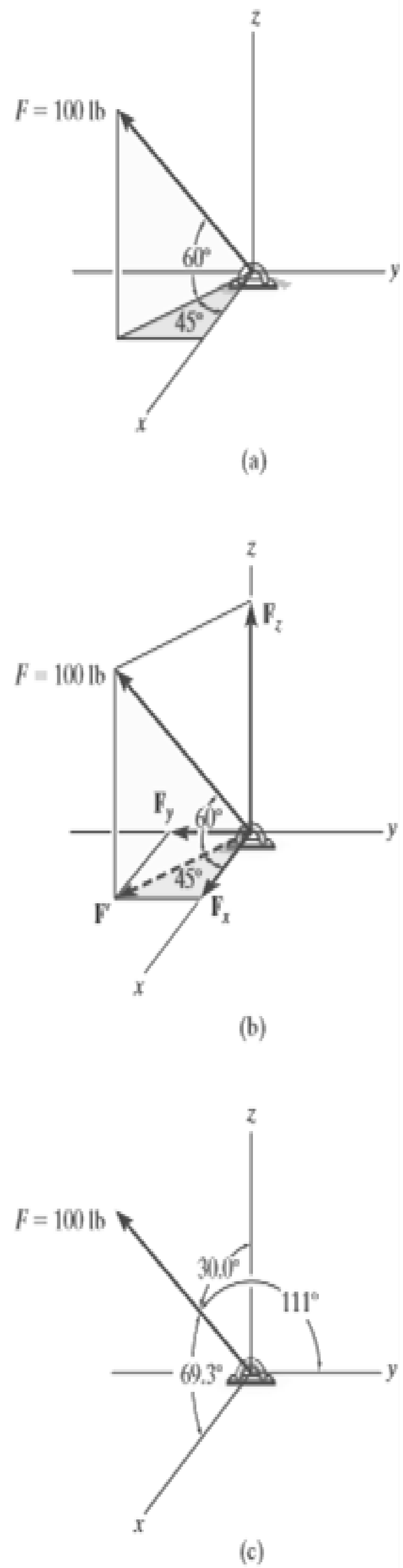


Fig. 2–32